Asynchronous games on Petri nets

Federica Adobbati

DISCo, Università degli studi di Milano-Bicocca

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Concurrent systems

- A user interacts with an environment
- The user knows the structure of the system
- The user may not observe everything
- The user has a goal (perform an action, avoid a state, ...)
- The environment is hostile or indifferent
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Applications
System control, model-checking, ...
Modelling language: Petri nets

1-safe distributed nets

- □ uncontrollable
- ■ controllable

Example: Can the user force the occurrence of transition $t_7$, by controlling only black transitions?
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Asynchronous game on Petri nets

Rules
- Whenever a transition is enabled, its owner can decide to 'fire' it.
- The Environment must guarantee the progress of the system.

Strategy
\[ \alpha : Mark(N) \to 2^K \]
Asynchronous game on concurrent game structure
**ATL (Alur, Henzinger, Kupferman 2002)**

**Syntax**

An ATL formula is one of the following:

- a proposition $p$
- $\neg \phi$ or $\phi_1 \lor \phi_2$
- $\langle\langle A \rangle\rangle X \phi$, $\langle\langle A \rangle\rangle G \phi$, $\langle\langle A \rangle\rangle F \phi$, or $\langle\langle A \rangle\rangle \phi_1 U \phi_2$, with $A$ set of players

$\phi, \phi_1, \phi_2$ ATL formulas

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**Example**

Is $\langle\langle \text{user} \rangle\rangle F p_7$ satisfied on this system?
Game on the unfolding
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- Run: a possible history of what happened in the net
- B-cut: a maximal set of pairwise concurrent places
Game on the unfolding

- Run: a possible history of what happened in the net
- B-cut: a maximal set of pairwise concurrent places
- Play: a run on the unfolding
- Strategy: $\alpha : Mark(N) \rightarrow 2^K$

The user has a **winning strategy** iff he wins all the plays that are consistent with the strategy.
Algorithm for the reachability of a target transition

Goal
The user reaches a target transition

General idea
- Recursive algorithm on the unfolding, with backtracking
- Generation of a finite prefix of the unfolding
- In every marking, the strategy selects all the controllable transitions that are ‘useful’ to reach the target
Mechanism of the algorithm

Unfolding_Exploration

From B-cut $\gamma$ add events until:
- Deadlock reached (the user tries to backtrack)
- Target fires (the environment tries to backtrack)
- Cycle detected (either no winning strategy or go to Explore_Cycle)

Explore_Cycle

- Add a transition concurrent with the cycle
- Restart with Unfolding_Exploration
Simulation of an execution
Simulation of an execution
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Simulation of an execution
Simulation of an execution
Critical example

Solved with backtracking
Ongoing works and future developments

- Correctness and complexity of the algorithm
- Partial observability
- Implementability of the strategy
Partial observability
Implementable strategies