Reachability in 2-parametric timed automata with one parameter is EXPSPACE-complete

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Automata have long been used to model computer systems and other kinds of program executions. However, for some computer systems, such as real time systems and embedded computer systems, the issue of time is an important one. Timed automata are a successful formalism to tackle modelisation and verification of such real time systems. Timed automata are automata extended with clocks evolving at the same rate, that can be reset, and that can be checked against clock expressions in order to enable or not certain transitions.

Furthermore, timing constraints in real time systems are sometimes dependent on the environment and factors hard to predict, therefore leaving some constraints under-specified in the timed automata model allows to check properties about environments, for instance in which kind of environment the system behaves as required. Such considerations led to the study of parametric timed automata. A parametric timed automaton is a timed automaton extended with a set of parameters, and whose guards are of the form $x \bowtie p$, $x \bowtie c$, where x is a clock, $c \in \mathbb{N}$ is a constant and p is a parameter, and $\bowtie \in \{<, \leq, =, \geq, >\}$, typically taking non-negative integers. A clock x in a parametric timed automaton is called parametric if there is at least one guard of the form $x \bowtie p$ for some parameter p.

Perhaps one of the most important problems in the study of a (parametric) timed automaton is the reachability problem: given an initial state, and an automaton $\mathcal{A}$ with a finite set of clocks X and a finite set of parameters P, does there exists an assignment of values to the parameters such that the automaton can perform a sequence of transitions in order to reach a certain final state? Rajeev Alur, Thomas A. Henzinger and Moshe Y. Vardi, in Proceedings of the twenty-fifth annual ACM symposium on Theory of computing, 1993, showed that the reachability problem is undecidable already for three parametric clocks. A few years ago, Bundala and Ouaknine proved that for two parametric clocks the problem is decidable in the presence of one parameter and also showed a

lower bound for the complexity class $\mathsf{P}^{\mathsf{NEXP}}$.

We investigate the complexity of the problem of reachability for parametric timed automata. Our main result is that reachability problem for two-parametric timed automata with one parameter is EXPSPACE-complete. Our contribution is two-fold.

For the lower bound, we design a framework for proofs of lower bounds in complexity. More explicitly, we develop a programming language with an EXPSPACE-complete halting problem that (1) itself can handily be encoded as runs in infinite-state systems that involve clocks or counters on the one hand, and that (2) is sufficiently expressive to encode bottleneck computations within the serializability view of EXPSPACE.

The technique we use is a general framework also for showing EXPSPACE-hardness of model checking succinct one-counter automata against fixed CTL formulas, and the reachability problem for succinct one-counter automata along with the bisimulation equivalence problem for succinct one-counter automata.

For the upper bound, we carefully partition a fictitious run into several carefully chosen subruns that allow us to prove that one can restrict oneself to a parameter evaluation of only doubly exponential magnitude. We hope that our upper bound approach sheds new light into attacking the long standing open problem of the reachability of parametric timed automata with two parametric clocks (and an arbitrary number of parameters).

Keywords: Parametric Timed Automata, One-Counter Machines, Reachability