Revisiting Synthesis for One-Counter Automata

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Counter automata are a classical model that extend finite-state automata with integer-valued registers. These have been shown to be useful in modelling complex systems, such as programs with lists and XML query evaluation algorithms. If the updates or tests applicable to the counter are *parameterized* by a set of integer-valued variables, they can also model simple software programs and their parameters. Despite their usefulness as a modelling formalism, it is known that two counters suffice for counter automata to become Turing powerful. In particular, this means that the most interesting questions are undecidable. To circumvent this, several restrictions of the model have been studied in the literature, e.g. reversal-bounded counter automata and one-counter automata. In this work we focus on an extension of the latter: one-counter automata with parametric updates and parametric (equality) tests.

One-counter automata are obtained by extending classical finite-state automata with a counter whose value can range over non-negative integers and be tested for zero. The updates and tests applicable to the counter can further be made parametric by introducing a set of integer-valued variables. Reachability for such automata is known to be decidable [7, 10]. Model-checking algorithms for such automata also exist for some specification logics [5, 6]. We revisit the parameter synthesis problem for such automata. That is, we ask whether there exists a valuation of the parameters such that all infinite runs of the automaton satisfy some omega-regular property. The problem has been shown to be encodable in a restricted one-alternation fragment of Presburger arithmetic with divisibility. In this work (i) we argue that said fragment of the logic is unfortunately undecidable. Nevertheless, by reduction to a class of partial observation games, (ii) we prove the synthesis problem is decidable. Finally, (iii) we give a polynomial-space algorithm for the problem if parameters can only be used in tests, and not updates, of the counter.

In [8], Lechner studied a synthesis problem for one-counter automata with parameters which asks whether there exists some value of the parameters such that all runs of the automaton satisfy a omega regular specification. By classical results, the problem is equivalent to asking whether all valuations make it so that all runs of the automaton satisfy some Büchi condition: they visit accepting states infinitely often. In that paper, Lechner proved that the Büchi condition can be decomposed into two reachability queries — intuitively a loop with an accepting state is reached and taken forever. She then goes on to reduce the resulting *synthesis reachability* problem to a fragment of Presburger arithmetic where a single alternation is allowed and a restricted divisibility(with variables) predicate can be used. In [3], said fragment was claimed to be decidable. However, in [9] the result from [3] was stated as being under review. We clarify the latter by proving the fragment is undecidable.

After invalidating the previous attempt at proving decidability of the synthesis reachability problem, we propose two alternatives. The first one based on the connection between counter automata and two-way automata; the second, on partial observation games. Specifically, assuming parameters are only present in counter tests, we provide small modifications to a recent construction due to Bollig et al. . [2] in order to obtain alternating two-way automata whose language is empty if and only if our synthesis reachability problem has a negative answer. Regarding partial observation games, we prove that the complement of the problem reduces to energy games with partial observation [1] where the initial credit is fixed [4, 11]. The partial observability allows us to force Eve, the protagonist, to faithfully simulate runs of the counter automaton under all possible valuations.

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References

[1] K. R. Apt and E. Grädel. *Lectures in game theory for computer scientists.* Cambridge University Press, 2011.

[2] B. Bollig, K. Quaas, and A. Sangnier. The complexity of flat freeze LTL. *Logical Methods in Computer Science*, 15(3), 2019.

[3] M. Bozga and R. Iosif. On decidability within the arithmetic of addition and divisibility. In V. Sassone, editor, *Foundations of Software Science and Computational Structures, 8th International Conference, FOSSACS 2005, Held as Part of the Joint European Conferences on Theory and Practice of Software, ETAPS 2005, Edinburgh, UK, April 4-8, 2005, Proceedings*, volume 3441 of *Lecture Notes in Computer Science*, pages 425–439. Springer, 2005.

[4] A. Degorre, L. Doyen, R. Gentilini, J. Raskin, and S. Toru'nczyk. Energy and mean-payoff games with imperfect information. In A. Dawar and H. Veith, editors, *Computer Science Logic, 24th International Workshop, CSL 2010, 19th Annual Conference of the EACSL, Brno, Czech Republic, August 23-27, 2010. Proceedings*, volume 6247 of *Lecture Notes in Computer Science*, pages 260–274. Springer, 2010.

[5] S. Göller, C. Haase, J. Ouaknine, and J. Worrell. Model checking succinct and parametric one-counter automata. In S. Abramsky, C. Gavoille, C. Kirchner, F. M. auf der Heide, and P. G. Spirakis, editors, *Automata, Languages and Programming, 37th International Colloquium, ICALP 2010, Bordeaux, France, July 6-10, 2010, Proceedings, Part II*, volume 6199 of *Lecture Notes in Computer Science*, pages 575–586. Springer, 2010.

[6] S. Göller, C. Haase, J. Ouaknine, and J. Worrell. Branching-time model checking of parametric one-counter automata. In *Foundations of Software Science and Computational Structures - 15th International Conference, FOSSACS 2012, Held as Part of the European Joint Conferences on Theory and Practice of Software, ETAPS 2012, Tallinn, Estonia, March 24 - April 1, 2012. Proceedings*, pages 406–420, 2012.

[7] C. Haase, S. Kreutzer, J. Ouaknine, and J. Worrell. Reachability in succinct and parametric one-counter automata. In *CONCUR 2009 - Concurrency Theory, 20th International Conference, CONCUR 2009, Bologna, Italy, September 1-4, 2009. Proceedings*, pages 369–383, 2009.

[8] A. Lechner. Synthesis problems for one-counter automata. In M. Bojanczyk, S. Lasota, and I. Potapov, editors, *Reachability Problems - 9th International Workshop, RP 2015, Warsaw, Poland, September 21-23, 2015, Proceedings*, volume 9328 of *Lecture Notes in Computer Science*, pages 89–100. Springer, 2015.

[9] A. Lechner. *Extensions of Presburger arithmetic and model checking one-counter automata.* PhD thesis, University of Oxford, 2016.

[10] A. Lechner, J. Ouaknine, and J. Worrell. On the complexity of linear arithmetic with divisibility. In *30th Annual ACM/IEEE Symposium on Logic in Computer Science, LICS 2015, Kyoto, Japan, July 6-10, 2015*, pages 667–676, 2015.

[11] G. A. Pérez. The fixed initial credit problem for partial-observation energy games is ack-complete. *Inf. Process. Lett.*, 118:91–99, 2017.