Partial Order Reduction for Trace Abstraction Refinement

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Motivation

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i := i-1
a[i] := 21
assert a[i] != 0
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a[j] := 7
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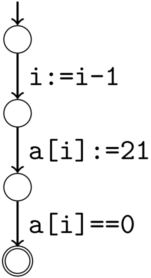
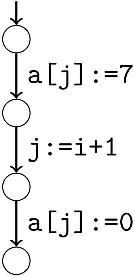
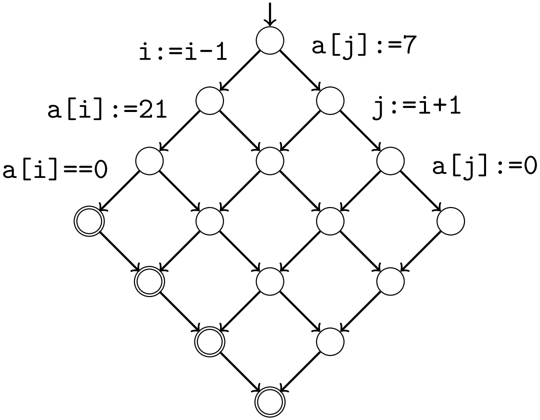




Motivation

Control Flow Automaton P:



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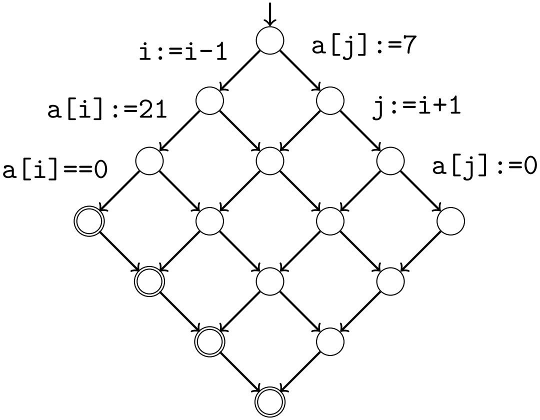

Goal: Prove P is *correct*

i.e. show all accepted *error traces* τ are *infeasible*:

$\{\textbf{true}\}\ \tau\ \{\textbf{false}\}$
valid Hoare triple

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iteratively build Floyd-Hoare automaton

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If $P \subseteq A$, then P is correct.

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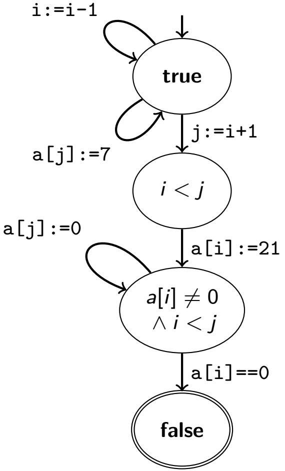

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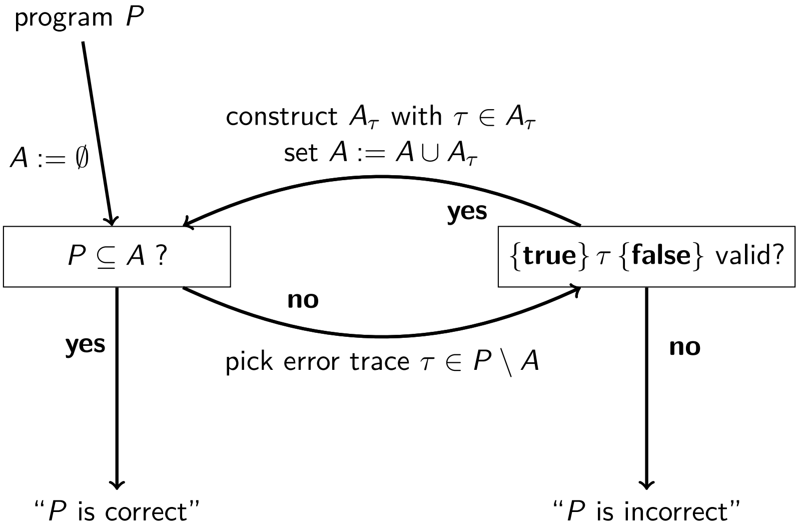

Partial Order Reduction

error traces:

τ_1 :	`i:=i-1`	`a[j]:=7`	`j:=i+1`	`a[i]:=21`	`a[j]:=0`	`a[i]==0`
τ_2 :	`i:=i-1`	`a[j]:=7`	`a[i]:=21`	`j:=i+1`	`a[j]:=0`	`a[i]==0`

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- Traces τ_1, τ_2 are *equivalent* ($\tau_1 \sim_I \tau_2$) iff

$$\tau_1 = \tau_2 \quad \textbf{or} \quad \begin{array}{c} \tau_1 = \rho a b \sigma, \tau_2 = \rho b a \sigma \\ \text{where } (a, b) \in I \end{array} \quad \textbf{or} \quad \exists \tau' \, . \, \tau_1 \sim \tau' \sim \tau_2$$

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- Goal: Only analyse one representative of each equivalence class!

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New proof criterion:

If $P \subseteq cl_I(A)$, then P is correct.

(for suitable commutativity relation I)

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Hence: Compute a (regular) reduction P' and check $P' \subseteq A$

- sufficient but (necessarily) incomplete
- more general than checking $P \subseteq A$

Concrete Commutativity

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Definition:

$$a \text{ and } b \text{ commute} \qquad \text{iff} \qquad [\![a]\!] \circ [\![b]\!] = [\![b]\!] \circ [\![a]\!]$$

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semantics of statement *a*
relation over program states

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For example:

$$[\![\mathtt{a[i]:=21}]\!] = \{\, (s, s') \mid s' = s\{\mathtt{a} \mapsto \mathit{store}(s(\mathtt{a}), s(\mathtt{i}), 21)\} \,\}$$
$$[\![\mathtt{j:=i+1}]\!] = \{\, (s, s') \mid s' = s\{\mathtt{j} \mapsto s(\mathtt{i}) + 1\} \,\}$$

Therefore

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Combined with Trace Abstraction Refinement by Cassez et al.

[1] Franck Cassez and Frowin Ziegler. "Verification of Concurrent Programs Using Trace Abstraction Refinement". In: *Logic for Programming, Artificial Intelligence, and Reasoning*. LNCS. Springer, Nov. 2015, pp. 233–248.

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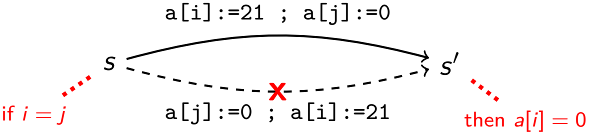

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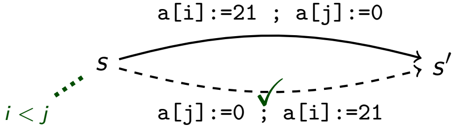

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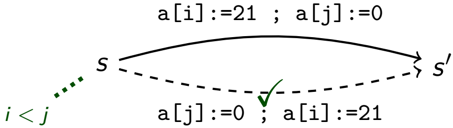

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[2] Patrice Godefroid. *Partial-Order Methods for the Verification of Concurrent Systems: An Approach to the State-Explosion Problem*. Springer, 1996.

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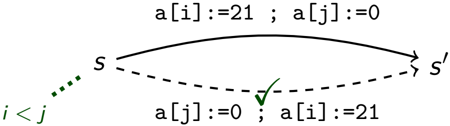

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- Here: state of Floyd-Hoare automaton A (i.e. a formula φ):

$$([\![a]\!] \circ [\![b]\!]) \cap (\varphi \times \textbf{true}) = ([\![b]\!] \circ [\![a]\!]) \cap (\varphi \times \textbf{true})$$

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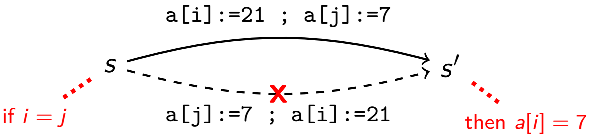

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[3] Tayfun Elmas, Shaz Qadeer, and Serdar Tasiran. "A Calculus of Atomic Actions". In: *Proceedings of the 36th Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*. POPL '09. New York, NY, USA: ACM, 2009, pp. 2–15.

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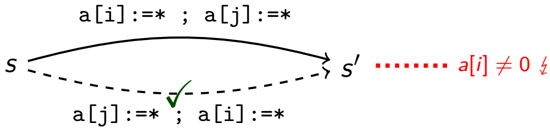

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  $\Rightarrow$ bound abstraction by proof candidate A

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`a[i]:=* with a[i]!=0 ; a[j]:=* with a[j]!=0`

$s \qquad s'$ ········ $a[i] \neq 0$ ✓

`a[j]:=* with a[j]!=0` ✓ `; a[i]:=* with a[i]!=0`

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Very general criterion:

$$P \subseteq cl_{concr}(A) \implies P \subseteq cl_{concr}(cl_{abstr}(A))$$
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- Theoretical complexity result

The End

Thank you for your attention.
Questions?