The Complexity of Bounded Context Switching with Dynamic Thread Creation

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Introduction

Example Program

```
global lock l;

main() {
  spawn handler();
  spawn main();
}

handler() {
  if * {
    a(0);
  } else {
    a(1);
  }
  unlock(l);
}
```

```
a(i) {
  if * {
    a(0);
  } elsif * {
    a(1);
  } else {
    lock(l);
  }

  // critical section:
  write(i);

  return;
}
```

Model Features

To model the behavior of the example, we need:

- Concurrent threads spawning dynamically during execution.
- A finite global memory, accessable by all threads.
- A local stack for each thread (e.g. to store procedure calls).
- Bound K on the number of context switches avoids undecidability.

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- Bound K on the number of context switches avoids undecidability.

For $K = 0$ safety is EXPSPACE-complete.

- Shown by Ganty and Majumdar (2012).

For $K \geq 1$ safety is EXPSPACE-hard and in 2EXPSPACE.

- Shown by Atig, Bouajjani, and Qadeer (2009).

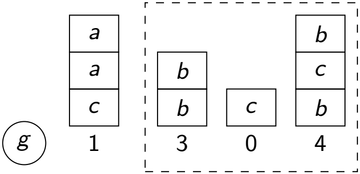
Our result closes the remaining complexity gap.

Dynamic Networks of Concurrent Pushdown Systems (DCPS)

Configurations

Consist of:

- A global state,
- an active thread with
  - a local stack
  - and a context switch number,
- and a bag of inactive threads.



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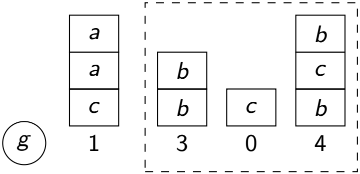
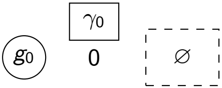


Initial configuration:

- Initial state g_0,
- an active thread with
  - only the initial symbol γ_0 on the stack
  - and context switch number 0,
- and an empty bag.



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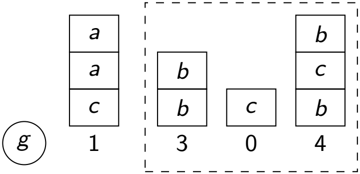

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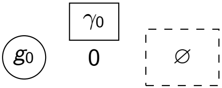
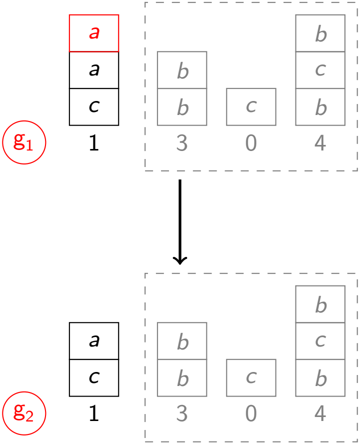


Pop, push, and spawn behavior is defined by transition rules.

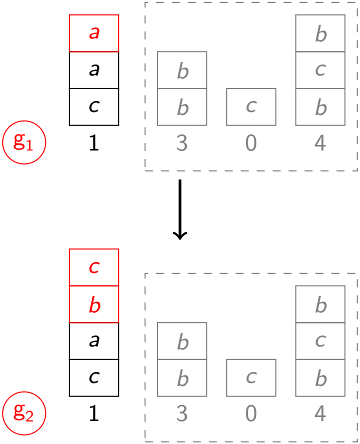
Pop Behavior

Consider transition rule $g_1|a \hookrightarrow g_2|\varepsilon$:



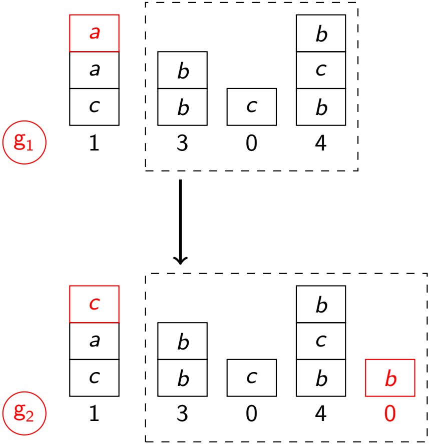
Push Behavior

Consider transition rule $g_1|a \hookrightarrow g_2|cb$:



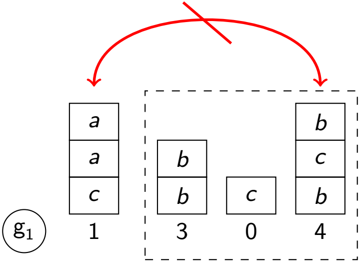
Spawn Behavior

Consider transition rule $g_1|a \hookrightarrow g_2|c \triangleright b$:



Context Switch Behavior

Consider the context switch bound $K = 3$:



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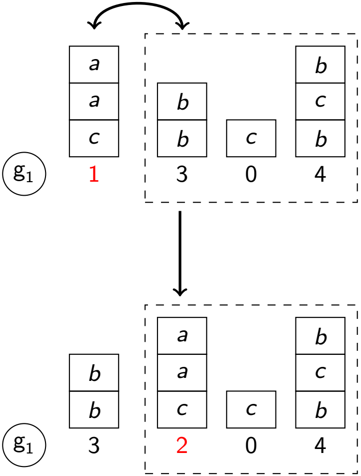

DCPS State Reachability

K-bounded state reachability problem for DCPS (SRP[K])

Input A DCPS $\mathcal{A}$ and a global state g

Question Is g K-bounded reachable in $\mathcal{A}$?

Main result

SRP[K] is 2EXPSPACE-hard for every $K \geq 1$.

Together with Atig, Bouajjani, and Qadeer (2009) this gives us:

Theorem

SRP[K] is 2EXPSPACE-complete for every $K \geq 1$.

2EXPSPACE Lower Bound

Proof Outline

The proof consists of a 3 step reduction:

Termination of triple-exponentially bounded counter programs

$\downarrow$

Termination of recursive net programs (RNP)*

$\downarrow$

Coverability for transducer-defined Petri nets (TDPN)*

$\downarrow$

SRP[1] for DCPS

*new model

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Transducer-Defined Petri Nets (TDPN)

Succinct representation of Petri nets:

Transducer-Defined Petri Nets (TDPN)

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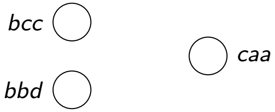
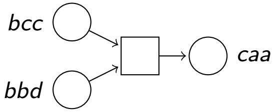
- Each Petri net place is assigned an address (word over an alphabet).



Transducer-Defined Petri Nets (TDPN)

Succinct representation of Petri nets:

- Each Petri net place is assigned an address (word over an alphabet).
- Petri net transitions correspond to vectors of such addresses.



$$\widehat{=} \begin{pmatrix} bcc \\ bbd \\ caa \end{pmatrix}$$

Petri net transition

Transducer-Defined Petri Nets (TDPN)

Succinct representation of Petri nets:

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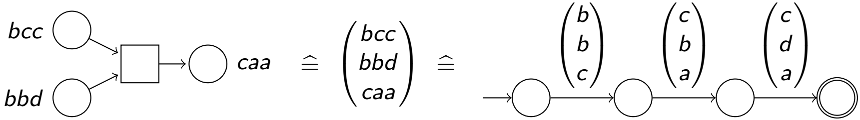
- Petri net transitions correspond to vectors of such addresses.
- These vectors are accepted by transducers.



Petri net transition

transducer run

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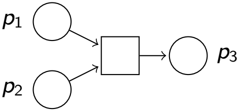
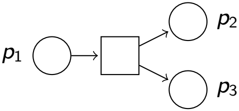


$$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} \in L(\mathcal{T}_{join})$$



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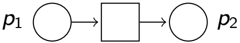

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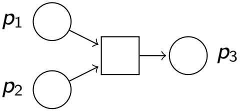
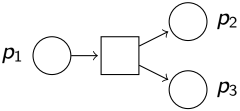
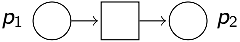






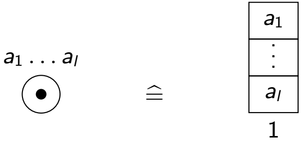
$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} \in L(\mathcal{T}_{join})$ $\quad \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} \in L(\mathcal{T}_{fork})$ $\quad \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} \in L(\mathcal{T}_{move})$

A TDPN consists of these three transducers plus two addresses:

- w_{init} for the initial marking:  w_{init} ,
- and w_{final} for the marking to cover:  w_{final}

Representing TDPN Tokens via DCPS

A token on place w corresponds to a thread with w on its stack:



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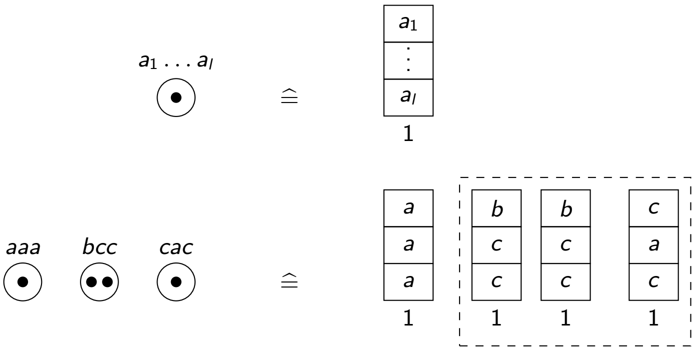
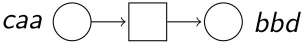


Simulating TDPN Transitions via DCPS: Read Stage



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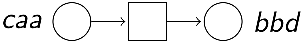

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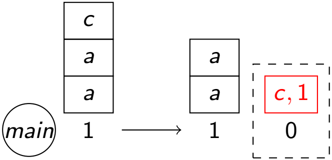

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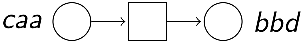

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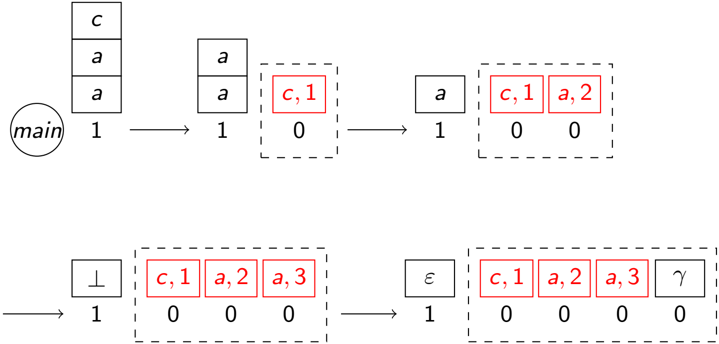

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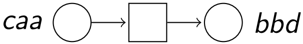

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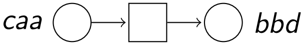

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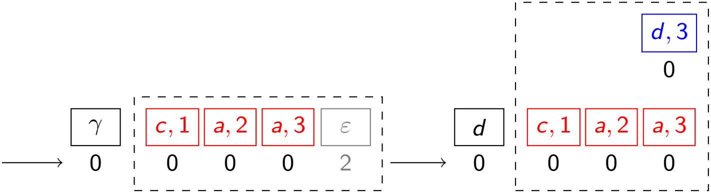

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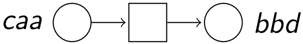

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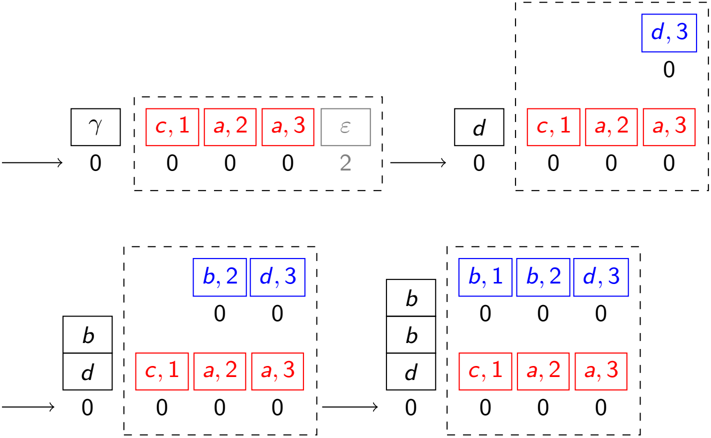

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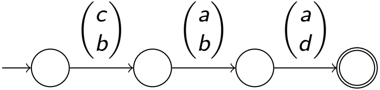

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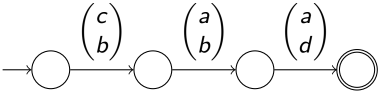

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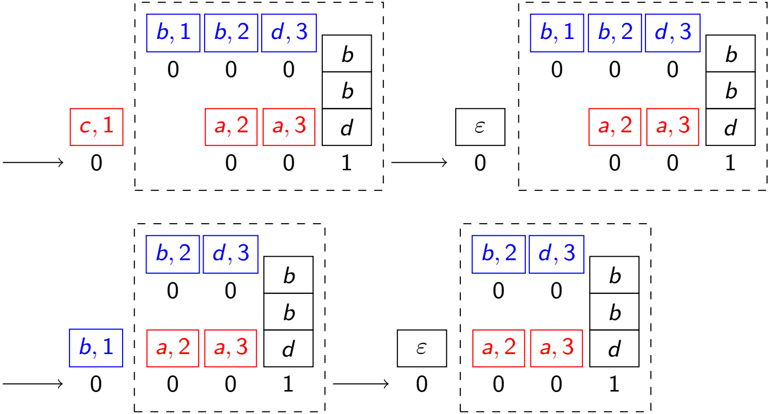

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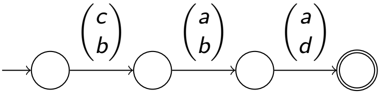

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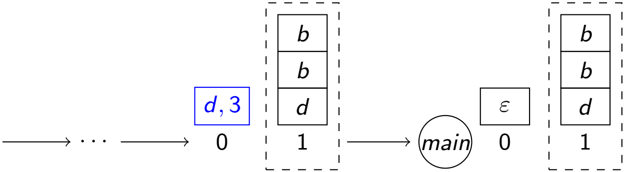
From here we continue until:



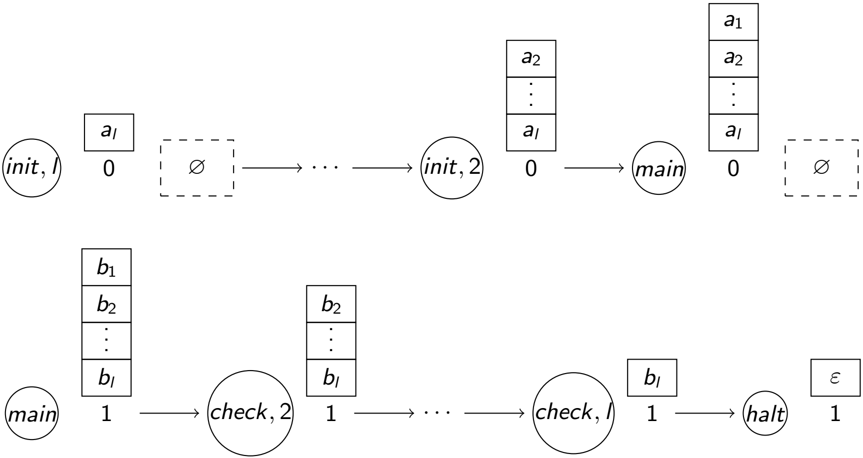
Simulating TDPN Transitions via DCPS

Transitions accepted by the other transducers require slight changes:

- $\mathcal{T}_{join}$ requires another thread's stack to be emptied,
- $\mathcal{T}_{fork}$ requires another new thread's stack to be guessed,
- and both of them need 3 sets of symbol threads in the bag.

Initial and Final Markings

Let $w_{init} = a_1 \dots a_l$ and $w_{final} = b_1 \dots b_l$.



Proof Outline

Termination of triple-exponentially bounded counter programs

Termination of recursive net programs (RNP)

Coverability for transducer-defined Petri nets (TDPN)

$\downarrow \checkmark$

SRP[1] for DCPS

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↓

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SRP[1] for DCPS

Instead of RNP, one could reduce from chain systems to TDPN.

- Introduced by Demri, Figueira and Praveen, 2013.

Original Lipton Construction

Due to Lipton, 1976.

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Shows EXPSPACE-hardness by reducing from bounded counter programs:

- Counters bounded by 2^{2^n} with n given in unary.
- Can the **halt** command be reached?

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Shows EXPSPACE-hardness by reducing from bounded counter programs:

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- Can the **halt** command be reached?

Reduce to reachability of the **halt** command in net programs:

- Counters cannot be tested for zero.
- To remedy this, keep a complement counter $\bar{x}$ for each counter x.
- Uphold invariant $x + \bar{x} = 2^{2^n}$.
- Test x for zero by checking that $\bar{x}$ is at maximum value.

Original Lipton Construction

We need to perform 2^{2^n} decrements on $\bar{x}$:

- Make use of the equality $2^{2^n} = 2^{2^{n-1}} \cdot 2^{2^{n-1}}$.
- Use two nested loops, each running $2^{2^{n-1}}$ times.
- Decrement a helper variable for each loop, test for zero at the end.
- Gives rise to nested zero tests, from level n down to 0.
- At level 0 decrement $2^{2^0} = 2$ times.

Original Lipton Construction

The program then consists of:

- $n + 1$ decrement subroutines for zero tests.
- $n + 1$ increment subroutines for initializing complement counters.
- $n + 1$ copies of each helper variable.

$$\begin{array}{ccc} \mathtt{dec}_n & \mathtt{inc}_n & y_n,\ z_n \\ \vdots & \vdots & \vdots \\ \mathtt{dec}_0 & \mathtt{inc}_0 & y_0,\ z_0 \end{array}$$

Adapted Lipton Construction

Get rid of duplicate code using recursion:

- Only a single recursive definition per subroutine.
- Implicitly use different variable copies depending on recursion depth.

`dec` `inc` y, z

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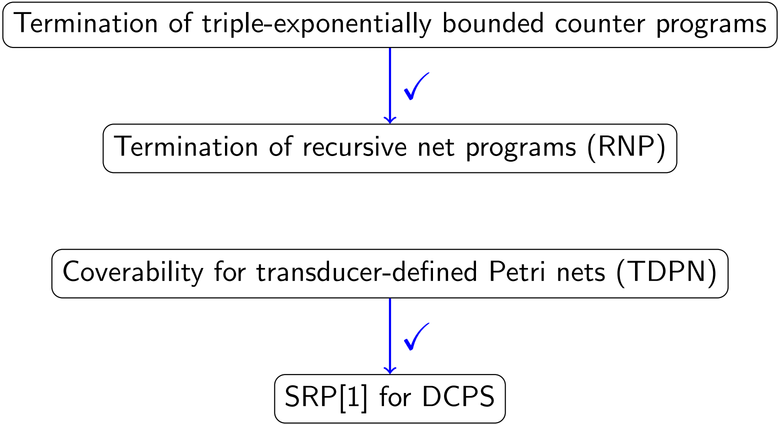
Recursive net programs (RNP) define a bound on their recursion depth.

- For EXPSPACE-hardness, use bound n.
- The bound 2^n would allow for triply exponential counter values.
- We can encode 2^n in poly(input) space, since n was given in unary.
- This then lifts the construction to 2EXPSPACE-hardness.

Proof Outline



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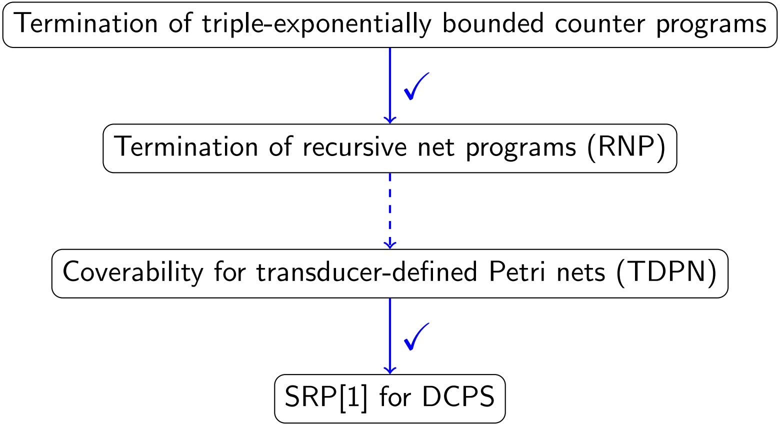

Simulating RNP via TDPN

For each possible recursion depth d from 0 to 2^n:

- one place per counter x,
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Use transitions to simulate the commands:

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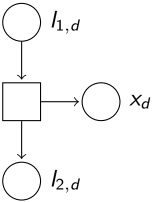

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Use transitions to simulate the commands:

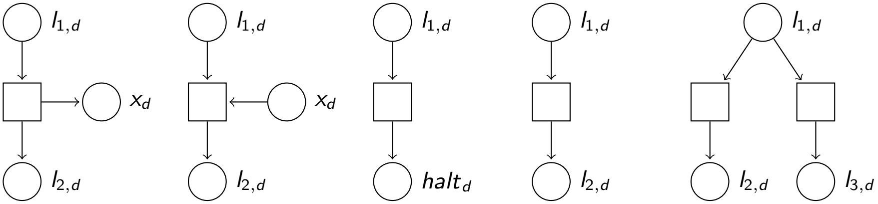
l_1 : **inc** x;
l_2 : . . .

l_1 : **dec** x;
l_2 : . . .

l_1 : **halt**;

l_1 : **goto** l_2;

l_1 : **goto** l_2 **or goto** l_3;



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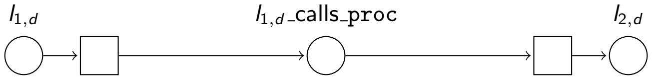
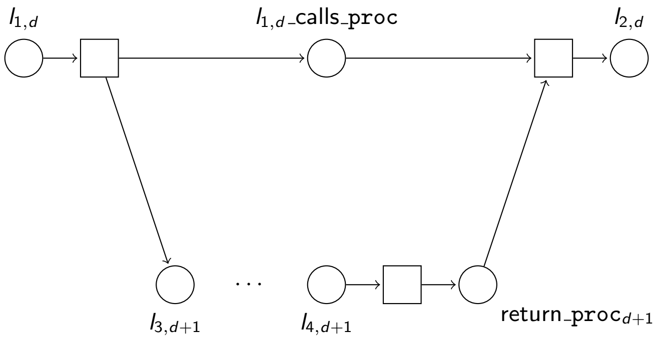


l_1 : **call** `proc`;

l_2 : ...

Simulating RNP via TDPN



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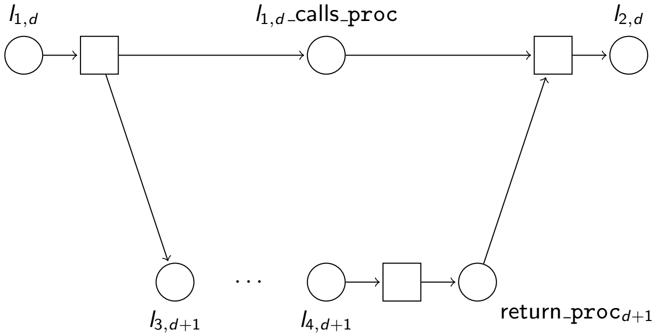
l_2 : ...

`proc`: l_3 : ...

⋮

l_4 : **return**;

Simulating RNP via TDPN



l_1 : **call** `proc`;
l_2 : ...

`proc`: l_3 : ...
⋮
l_4 : **return**;

- Define w_{init} as the first line in the program at depth 0.
- Define w_{final} as the auxiliary halting place at depth 0.

Succinct Representation via Transducers

Use binary addresses $w = u.v$ for places:

- u: Role, i.e. which line, counter, or auxiliary place it is.
- v: Binary representation of recursion depth d.

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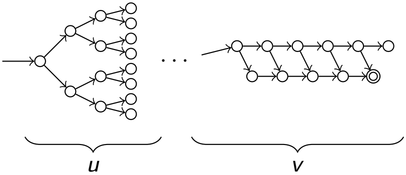
Construct transducers with polynomially many states:

- There are only polynomially many possibilities for triples of u.
- Comparing binary encodings of d takes polynomial space.



Proof Outline

Termination of triple-exponentially bounded counter programs

$\downarrow$ ✓

Termination of recursive net programs (RNP)

$\downarrow$ ✓

Coverability for transducer-defined Petri nets (TDPN)

$\downarrow$ ✓

SRP[1] for DCPS

Conclusion

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Together with previously known results we have shown:

Theorem

SRP[K] is EXPSPACE-complete for $K = 0$,
and 2EXPSPACE-complete for every $K \geq 1$.

There are no remaining complexity gaps!

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We introduced two new natural models, RNP and TDPN.

- Their 2EXPSPACE-complete problems are of independent interest.

Our hardness result can even be applied to an adjacent model:

- Replicated finite-state programs (Kaiser, Kroening, and Wahl, 2010).

Ongoing Work

Study problems for DCPS related to liveness verification:

- Non-termination.
- Fair non-termination (distinguish threads with different make-up).
- Non-starvation (distinguish all threads).

Thank you for your attention!

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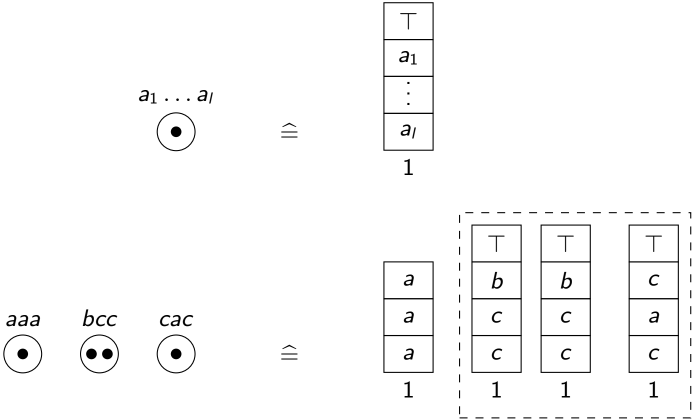
Locking Inactive Threads



Lifting to 2EXPSPACE

We used $2^{2^d} = 2^{2^{d-1} \cdot 2} = \left(2^{2^{d-1}}\right)^2 = 2^{2^{d-1}} \cdot 2^{2^{d-1}}$.

- This means from one level to the next the bound gets squared.

$$(\cdots(2\overbrace{{}^{2})^{2}\cdots)^{2}}^{n\text{-times}} = 2^{2^n}$$

$$(\cdots(2\overbrace{{}^{2})^{2}\cdots)^{2}}^{2^n\text{-times}} = 2^{2^{2^n}}$$

Details of Known Results

Ganty and Majumdar (2012) consider threads running to completion.

- We can ensure that threads empty their stack in our model.
- This allows us to use their EXPSPACE-completeness result for $K = 0$.

Atig, Bouajjani, and Qadeer (2009) consider a slightly different DCPS:

- Each thread spawns with its parents cs-number plus 1.
- We can simulate our model in theirs using 2 more contex switches.
- Reduces our SRP[K] to their SRP[$K + 2$].
- This allows us to use their 2EXPSPACE-membership result.

Succinct Representation via Transducers

Use binary addresses $w = u.v$ for places:

- u: Role, i.e. which line, counter, or auxiliary place it is.
- v: Binary representation of recursion depth d.

Let the size of the RNP be h, the number of lines of code.

- Each counter appears in at least one line.
- Each line only needs at most one auxiliary place.
- Thus, the number of possibilities for u is linear in h.

Make the transducers distinguish each possible triple (pair) of prefixes u:

- Considering triples adds an exponent of 3, still poly in h.

Succinct Representation via Transducers

The recursion depth d changes by at most 1 at a time.

- Transducers have to check for equality or off-by-one on postfixes v.
- These checks require space linear in the number of bits.
- Since the maximum for d is 2^n, v has $\log(2^n) = n$ bits.

The triple (pair) of prefixes u tells us how the depths are related.

- Connect the paths for u with the appropriate checks at the end.