# Revisiting Synthesis for One-Counter Automata

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*(Joint work with Guillermo A. Perez)*

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Universiteit Antwerpen

# Outline

# One-Counter Automata

```
n=5
n= max(0,n-5)
if n=0:
    while (n<10):
      n=n+1
    n= n+100
else:
    return false
```

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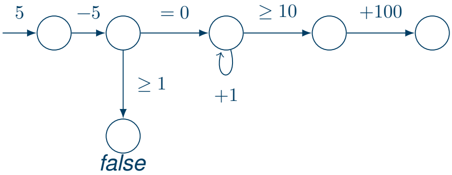


- Configurations: $(q, c), c \geq 0$;

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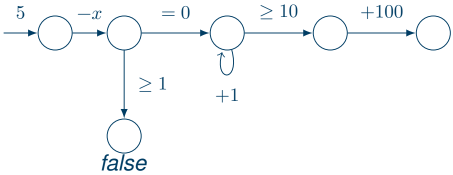


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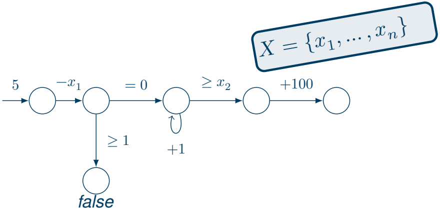

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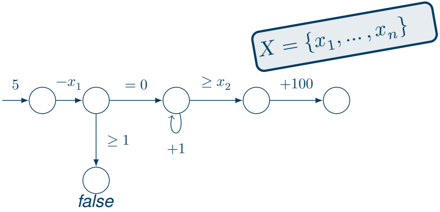


**Definition (<S>OCA<P>)**

$\mathcal{A} = (Q, T, \delta, q_{in}, X) \quad \delta : T \to Op$

$Op :=$

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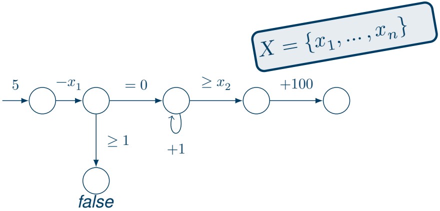


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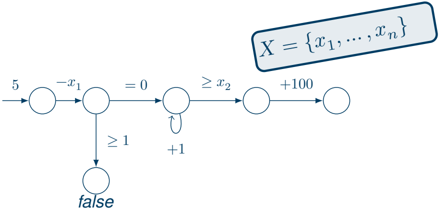


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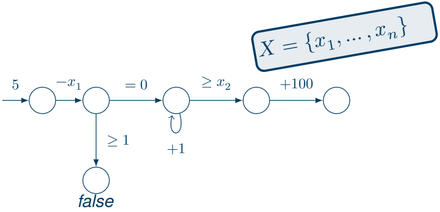


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Non-parametric: $X = \emptyset$

# One-Counter Automata

| Models | | *CU* | *PU* | *ZT* | *PT* |
|---|---|---|---|---|---|
| Non-Parametric | SOCA | ✓ | ✗ | ✓ | ✗ |
| Parametric | OCAPT | $\{-1, 0, 1\}$ | ✗ | ✓ | ✓ |
| | SOCAP | ✓ | ✓ | ✓ | ✓ |

# Decision Problems

**Non-Parametric:**

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**Reach**

$\exists \rho$ such that

$(q_{in}, 0) \xrightarrow{\rho} q_f$

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For all infinite $\rho$,

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$Op = CU \cup ZT$;     UnivReach: all infinite paths reach $q_f$;

$\neg$Reach := two Reach queries (coNP);

Hardness from reduction from co-subsetsum.

**Proposition**

*The UnivReach problem for SOCA is coNP-complete.*

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**SYNTHREACH**
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# PAD & Extensions

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**Theorem (Lechner-Ouaknine-Worrell'15)**

*EPAD is decidable in NEXPTIME.*

# Outline

1. One-Counter Automata(Parametric) and Synthesis Problem
2. Previous Approach
3. Approach with Alternating Two Way Automata for a subclass
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# $\forall\exists_R$PAD & Undecidability

- $\forall\exists_R$PAD $:= \forall z_1 \ldots \forall z_n \exists x_1 \ldots \exists x_m \varphi(\vec{x}, \vec{z})$

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$\neg(a \mid b) \equiv b = aq + r$ where, $0 < r < b$.

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Using techniques from Bozga-Iosif'05 and Lechner'15:

$\forall\exists_R\text{PAD}^+ \iff \forall\exists_R\text{PAD}$

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Undecidable!!

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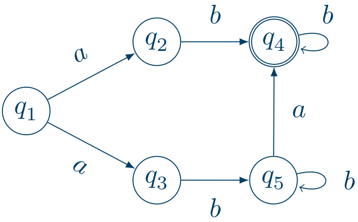

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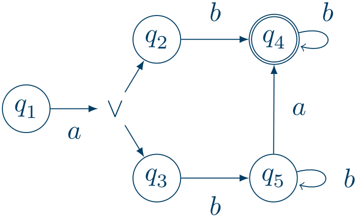

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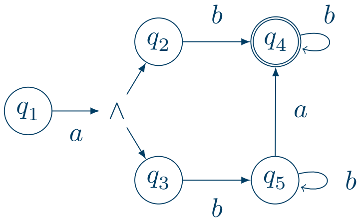

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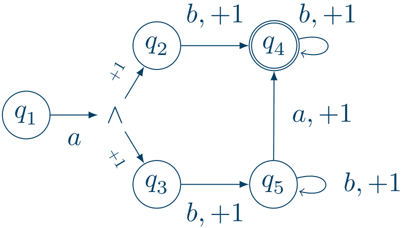

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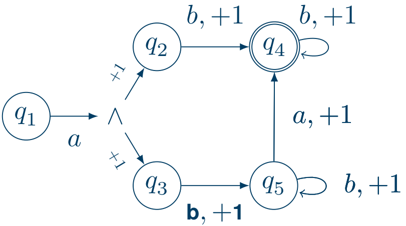

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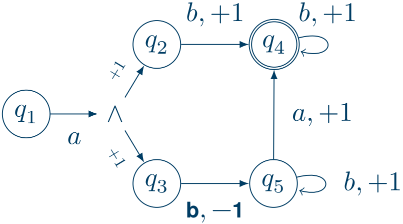

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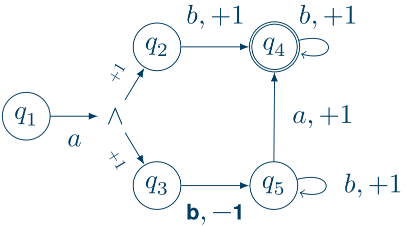


**Theorem (Serre'06)**

*The non-emptiness problem for A2As is in PSPACE.*

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**Proposition (Similar Idea to Bollig-Quaas-Sangnier'19)**

*For every OCAPT $\mathcal{A} \Rightarrow$ an A2A $\mathcal{T}$ of poly-size such that, it accepts words corresponding to valuations for which SynthReach is true.*

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- Accept the reaching runs

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Valuation to Words:

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$V : X \to \mathbb{N}; \qquad \Sigma = X \cup \{\Box\};$

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$\square$

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$\square\square\square x_1 \square x_2 \square^\omega$

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$V : X \to \mathbb{N}; \qquad \Sigma = X \cup \{\Box\};$

$\Box\Box\Box x_1 \Box x_2 \Box^\omega \Rightarrow x_1 \mapsto 2, x_2 \mapsto 3;$

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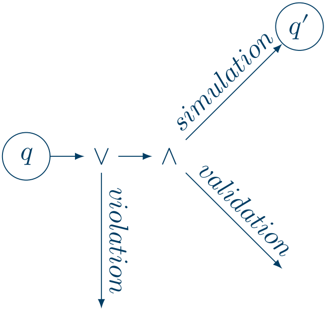

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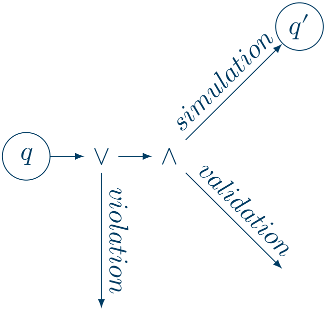


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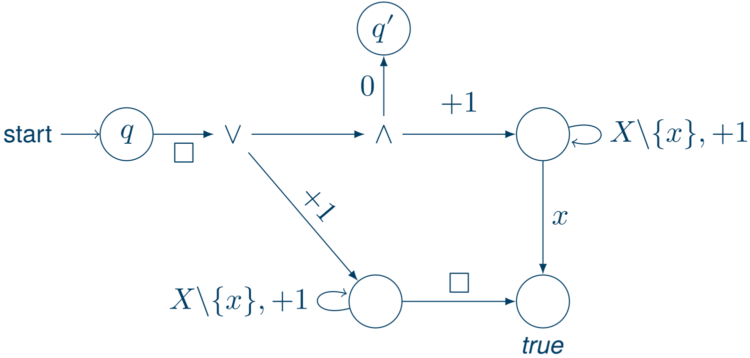

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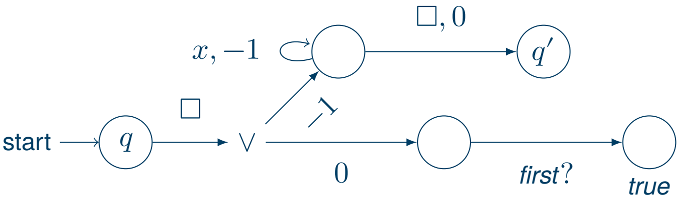

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$$q \xrightarrow{op_1} q_1 \Rightarrow (q, \Box, \mathcal{T}_1)$$

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**Lemma**

*If SYNTHREACH is true then we can bound the valuation to be exponential and thus poly size in succinct encoding.*

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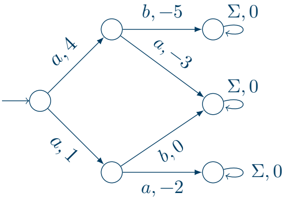
Guess the valuation $\Rightarrow$ UnivReach of SOCA $\Rightarrow$ $\text{NP}^{\text{NP}}$!!

# Outline

1. One-Counter Automata (Parametric) and Synthesis Problem
2. Previous Approach
3. Approach with Alternating Two Way Automata for a subclass
4. Approach with Partial Observation Games

# Partial Observation Energy Game



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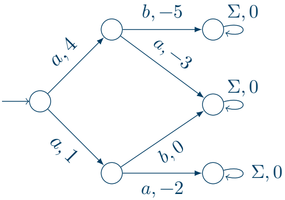


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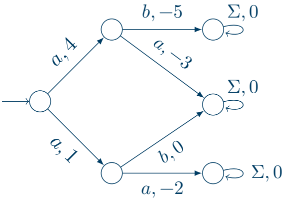


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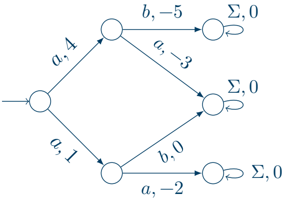


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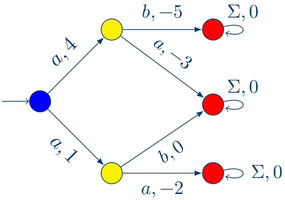


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**Lemma**

*For every SOCAP $\mathcal{A}$ $\Rightarrow$ an POEG $\mathcal{G}$ such that for all valuations $V$ there exists a reaching run of $\mathcal{A}$ iff Eve has a winning strategy in $\mathcal{G}$.*

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**Theorem**

*SynthReach problem for SOCAP is decidable.*

# Decision Problems

**Non-Parametric:**

**REACH**

$\exists \rho$ such that $(q_{in}, 0) \xrightarrow{\rho} q_f$

[NP-complete]

**UNIVREACH**

For all infinite $\rho$, $(q_{in}, 0) \xrightarrow{\rho} q_f$

[coNP-complete]

**Parametric:**

**PAR-REACH**

$\exists V(X)$ such that $\exists \rho$, $(q_{in}, 0) \xrightarrow{\rho}_V q_f$

[in NEXPTIME]

**SYNTHREACH**

$\exists V(X)$ such that for all infinite $\rho$, $(q_{in}, 0) \xrightarrow{\rho}_V q_f$

[Decidable]

# Conclusion

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  Approach with Multi Energy Game with resets and transfers- Better Complexity?