Local first order logic for distributed algorithms

MOVEP 2020

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Joint with B. Bollig and A. Sangnier
This Talk:

- Consensus problem
- Local first order logique
- Results
The Consensus Problem
The Consensus Problem

Input
The Consensus Problem

Input

Output
The Consensus Problem

Input

Output

Crashed
Expressiveness: IO-FO

Models  |  Syntax
Expressiveness: IO-FO

Models

\[ \mathcal{A} = (A, \ell, I, O) \]

Where

\( A \) is a finite set

\( \ell : A \rightarrow \Sigma \) Labeling

\( I : A \rightarrow \mathbb{N} \) Data values at input

\( O : A \rightarrow \mathbb{N} \) Data values at output

Syntax
Expressiveness: IO-FO

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Syntax
\[ \varphi ::= x = y \mid \varphi \lor \varphi \mid \neg \varphi \mid \exists x. \varphi \mid a(x) \]
Expressiveness: IO-FO

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\[ \varphi_{con} = \exists x.(x_I \sim O x \land \forall y.(q_f(y) \Rightarrow x_I \sim O y)) \]
Decidability: Loc-IO-FO

Issue: Satisfiability undecidable for IO-FO

Given a formula,
decide if it has a model

$$\psi_1$$
Decidability: Loc-IO-FO

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Top level

Bottom level

All data comparison appear at bottom level

Each $\psi$ has exactly one free variable whose is involved in all data comparison
Decidability: Loc-IO-FO

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All data comparison appear at bottom level

Each $\psi$ has exactly one free variable whose is involved in all data comparison

$\varphi = \forall x. \exists y z. x \sim_I y \land y \sim o z \land a(z) \notin \text{Loc-IO-FO}$
Decidability: Loc-IO-FO

**Issue:** Satisfiability undecidable for IO-FO

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Each $\psi$ has exactly one free variable whose involvement in all data comparison

$$\varphi = \forall x. \exists yz. x I\sim I y \land y o\sim_o z \land a(z) \notin \text{Loc-IO-FO}$$

$$\varphi_{con} = \exists x. [x I\sim O x \land \forall y. (q_f(y) \Rightarrow x I\sim O y)] \in \text{Loc-IO-FO}$$
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$\varphi = \forall x. \exists y z. x I \sim_I y \land y o \sim_o z \land a(z) \notin \text{Loc-IO-FO}$

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Is sat for Loc-IO-FO decidable?
Related Works

- Locality in a new concept
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- Closest paper:

  On Finite Satisfiability of Two-Variable First-Order Logic with Equivalence Relations
  Kieronski, Emanuel and Tendera, Lidia

  $\text{FO}_2[\Sigma, \sim_1, \sim_2]$
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\[
\text{FO}_2[\Sigma, \sim_1, \sim_2]
\]

- Loc-IO-FO is an extension:
  - Two-variable FO can be made local (Scott NF)
  - The two equivalence relations interact
(Partial) Results

Existential fragment =
DEF: Only existential quantifier on top level
(Partial) Results

Exsitential fragment =
DEF: Only existential quantifier on top level

THM: Sat for the existential fragment is PSAPCE-complet
(Partial) Results

Exsitential fragment:
DEF: Only existential quantifier on top level

THM: Sat for the existential fragment is PSAPCE-complete

Sat for Loc-IO-FO is NEXPTIME-Hard

Lower bound: